Locally s-arc transitive graphs

Jozef van Bon
Dipartimento di Matematica e Informatica, Università della Calabria, 87036 Rende, Italy
e-mail: jozef.vanbon@unical.it

A G-graph is a connected simple graph Δ, together with a subgroup $G \leq \mathrm{Aut}(\Delta)$. The vertex set of Δ is denoted by $V\Delta$ and the edge set is $E\Delta$. The stabilizer G of a vertex $X_0 \in V\Delta$ is denoted by G_{x_0}. An *s-arc* emanating from a vertex $x_0 \in V\Delta$ is a path $(x_0, x_1, \ldots, x_s)$ with $x_{i-1} \neq x_{i+1}$ for $1 \leq i \leq s-1$.

A G-graph Δ is called *locally s-arc transitive* if for any vertex x_0 the stabilizer G_{x_0} is transitive on the set of s-arcs emanating from x_0, and is called *locally finite* if all vertex stabilizers are finite groups. For an 1-arc (x_1, x_2) of Δ the triple $(G_{x_1}, G_{x_2}; G_{x_1,x_2})$ is called the *vertex stabilizer amalgam* with respect to that arc.

Let Δ be a locally finite G-graph with $s \geq 1$. Then G has at most two orbits on the vertex set $V\Delta$ and is transitive on the set of edges $E\Delta$. Therefore, the vertex stabilizer amalgam does not depend on the choice of the edge, and describes the graph and group locally. In general, not much can be said about the vertex stabilizer amalgam of a locally finite G-graph with $s \geq 1$. However, when $s \geq 6$, it known by [1]that vertex stabilizer amalgam has to be a weak BN-pair.

In this talk, we will give a brief overview of the theory of locally finite and locally s-arc transitive graphs, and will discuss some recent results on the structure of their vertex stabilizer amalgams when $s \geq 4$.

References

[1] J. van Bon and B. Stellmacher, Locally s-transitive graphs, *J. Algebra* **441** (2015) 243–293.